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Sensitivity Analysis Using Sobol 'Variance-Based Method on the Haverkamp Constitutive Functions Implemented in Richards' Water Flow Equation

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ABSTRACT

Richards' equation was approximated by finite-difference solution and implemented in FORTRAN to simulate water infiltration profile of yolo light clay. The simulation was successfully validated by published data of Philip's semi-analytical solution. Global sensitivity analysis using Sobol' variance-based method was also coded in FORTRAN and implemented to study the effect of parameter uncertainty on model output variability. Sobol' sequences were used to generate quasi-random numbers to study the effect of every possible combination of different input parameters' values, based on each parameter's uncertainty range on model outputs. First order sensitivity index (S_i) and total effect index (S_{T_i}) were estimated based on quasi-Monte Carlo estimators. Various statistical parameters, coded in FORTRAN, such as kurtosis, skewness, 95% confident intervals, etc. were used to provide a better understanding and description of the model outputs. Results found parameter constants (β, B) and saturated volumetric water content (θ_s) of Haverkamp constitutive functions to be dominant parameters with a combined 93% of model variability which could be explained by these parameters. The total effect index for every parameter was found to be greater than the first order effect index. In addition, global sensitivity analysis tool was able to generate informative sensitivity indicators and a good statistical description compared to the local sensitivity tool.

Keywords: **First order index, global sensitivity analysis, Haverkamp constitutive functions, Richards' equation, Sobol' variance-based method, total effect index, uncertainty analysis**

INTRODUCTION

Sensitivity analysis can be broadly categorized into local sensitivity analysis and global sensitivity analysis. Local sensitivity analysis, that is, normally known as one-at-a-time (OAT) measure, is carried out by varying single input parameters of interest and keeping other parameters at constant value in order to study model output. This is commonly used by various researchers (Vereecken *et al.* 1990; De Roo and Offermans 1995; Jhorar *et al.* 2002). The obvious problem of this method

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is that interaction between various input parameters cannot be identified when two or more input parameters vary simultaneously.

An alternative to local sensitivity analysis is the global sensitivity analysis, of which there are several varieties ranging from a qualitative screening method to quantitative technique (Sobol' 1990; Morris 1991; Campolongo *et al*. 2007). One type of global sensitivity analysis is the variance-based Sobol' method (Sobol' 1990). Its main purpose is to quantify the variance contribution of input parameters to the unconditional variance of model output. This is important in determining input parameters with the greatest influence for parameter prioritization, and input parameters with the least influence for parameter fixing (Saltelli *et al.* 2008). It has been applied on various models, for example, in Soil and Water Assessment Tool (SWAT) by Nossent *et al.* (2011), inhalation dose model by Avagliano and Parrella (2009), CERES-EGC model by Drouet *et al*. (2011), and Sea Level Affecting Marshes Model (SLAMM 5) by Chu-Agor *et al*. (2011).

Richards' equation (Richards 1931) is widely used to predict water movement in variably saturated soils. This equation has important applications in hydrology, meteorology, agronomy, environmental protection, and other soil-related fields (Pachepsky *et al.* 2003). In the field of environmental protection, an accurate prediction of water movement will allow for estimation of the movement of pollutants in contaminant transport models (Šimůnek and Bradford 2008). It has also been used in geotechnical and geo-environmental engineering to predict unsaturated flow of water in unsaturated soils (Barari *et al.* 2009). The equation is a combination of both Darcy's law and continuity equation, and is known for its non-linearity property (Caviedes-Voullième *et al*. 2013). This is attributed to the relations between soil water content on soil water pressure head and hydraulic conductivity (Feddes *et al*. 1988). Equations which are used to govern the relations are commonly known as a constitutive function, such as the Haverkamp constitutive function (Haverkamp *et al.* 1977). Namin and Boroomand (2012) state that Richards' equation numerical solution strategy is still a subject to research.

According to Mishra (2009), there are basically three elements of uncertainty: (1) uncertainty of characterization, i.e. distribution types of uncertainty model inputs; (2) uncertainty propagation that involves translating the uncertainty in model inputs to the uncertainty in model outputs; and (3) uncertainty importance to determine the influential parameters. Uncertainty analysis is superior to sensitivity analysis because it includes both uncertainty characterization and propagation, whereas sensitivity analysis includes only uncertainty importance. It is common that both sensitivity analysis and uncertainty analysis coupled in practice and only named as sensitivity analysis (Saltelli and Annoni 2010). We reserved the term of sensitivity analysis for both analyses. Moreover, the variance-based method has been listed in the US Environmental Protection Agency's (EPA 2009) list of attributes as the preferred sensitivity analysis method. It is claimed to be robust and independent, irrespective of model assumptions (Saltelli *et al*. 2000).

In this study, Haverkamp constitutive functions implemented in Richards' equation were subjected to sensitivity analysis by the Sobol' variance-based method to determine the effect of input parameter uncertainty on model outputs. The numerical solution on Haverkamp constitutive functions and Richards' equation were implemented and tested with a case study from Haverkamp *et al*. (1977). The uncertainty range of input parameters was established based on significant digits approximation. The variance-based sensitivity analysis was carried out on Haverkamp constitutive functions to determine the first order sensitivity index (S_i) and total effect index (S_{T_i}). The first order sensitivity index (S_i) indicates the main effect output variance of input parameter, X_i, that could be reduced if it could be fixed to a specific value. The S_{T_i} indicates the sum that includes first order index (S_i), second order index (S_{ij}, S_{ik}, and $S_{i...q}$, where q is the total number of input parameters in the model), third order index and so on. It is generally acceptable to determine only S_i and S_{T_i} (Fox *et al*. 2010). The variance-based method was also compared to the local sensitivity analysis results.

Sobol' Variance-Based Method as Global Sensitivity Analysis Tool

Model outputs (Y) is a function of input parameters (X_1, X_2, X_3,...,X_q), and thus, this relation can be written as follows:

$$Y = f(X_1, X_2, X_3, \ldots, X_q) \quad (1)$$

In the variance-based method, total unconditional variance, $V(Y)$, in Eq. (1), can be decomposed into partial variances of increasing dimensionality (Sobol' 1990):

$$V(Y) = \sum_i^q V_i + \sum_i^q \sum_{j>i}^q V_{ij} + \cdots + V_{12\ldots q} \quad (2)$$

where $\sum_i^q V_i$ is the sum of partial variances that include main effects of each input parameter, $\sum_i^q \sum_{j>i}^q V_{ij}$ includes all the partial variances of interaction of two input parameters and so on.

Dividing Eq. (2) by total unconditional variance, it becomes:

$$\sum_i^q S_i + \sum_i^q \sum_{j>i}^q S_{ij} + \cdots + S_{12\ldots q} = 1 \quad (3)$$

where $\sum_i^q S_i = \frac{\sum_i^q V_i}{V(Y)}$ is the sum of first order indices, $\sum_i^q \sum_{j>i}^q S_{ij} = \sum_i^q \sum_{j>i}^q V_{ij} / V(Y)$ is the sum of second order indices and so on. Eq. (3) is only valid when all input parameters are independent (orthogonal) of each other. Hence, first order index (S_i) and second order index (S_{ij}) for each input parameter are given as follows (Sobol' 1990):

$$S_i = \frac{V_i}{V(Y)} \quad (4)$$

$$S_{ij} = \frac{V_{ij}}{V(Y)} \quad (5)$$

Due to the ratio of partial variances (e.g. V_i, V_{ij}, etc) to total variance ($V(Y)$), all the sensitivity indices are scaled between 0 and 1 interval. When the summation of all first order indices gives unity, that is, $\sum_i^q S_i = 1$, the model is known as additive, that is, without any interaction effect. Hence, 1- $\sum_i^q S_i$ indicates interaction effects that could either be one or a combination of second order, or higher order.

The total effect index (S_{T_i}) for each input parameter is given by:

$$S_{T_i} = S_i + \sum_{i \neq j} S_{ij} + \sum_{i \neq j \neq l} S_{ijl} + \cdots \quad (6)$$

For instance, if $q = 3$, total effect index would be given by:

$$S_{T_1} = S_1 + S_{12} + S_{13} + S_{123} \quad (7)$$

where S_1 is first order index of input parameter 1, S_{12} is second order index of interaction effect between input parameters 1 and 2, S_{13} is second order index of parameters 1 and 3, and S_{123} is third order index of interaction effect between input parameters 1, 2 and 3. The same method is applied for decomposition of S_{T_2} and S_{T_3} total effect index. Since S_{T_i} includes first to higher order relating to input parameter i, S_{T_i} - S_i indicates only interaction effect that only account for second and higher order indices.

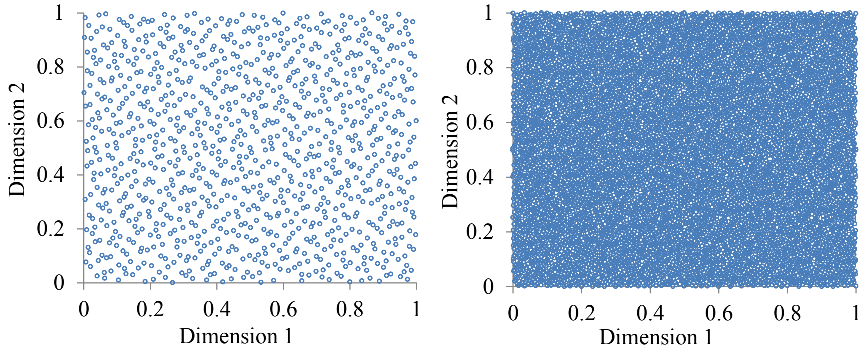
First order sensitivity index and total effect index were estimated by quasi-Monte Carlo estimators (Saltelli *et al*. 2010):

$$S_i = \frac{\left[(1/N)\sum_{m=1}^{N}\left(y_A^{(m)}\right)^2 - f_o^2\right] - \left[(1/2N)\sum_{m=1}^{N}\left(y_B^{(m)} - y_{C_i}^{(m)}\right)^2\right]}{(1/N)\sum_{m=1}^{N}\left(y_A^{(m)}\right)^2 - f_o^2} \quad (8)$$

$$S_{T_i} = \frac{(1/2N)\sum_{m=1}^{N}\left(y_A^{(m)} - y_{C_i}^{(m)}\right)^2}{(1/N)\sum_{m=1}^{N}\left(y_A^{(m)}\right)^2 - f_o^2} \quad (9)$$

Both $y_A^{(m)}$ and $y_B^{(m)}$ are model outputs, as shown in Eqs. (8) and (9). Sobol' quasi-random sequences were used to generate two sets of data, that is, matrix A and B corresponding to model outputs of $y_A^{(m)}$ and $y_B^{(m)}$, and these dataset were confined between 0 and 1. The advantage of Sobol' quasi-random sequence is that it produced good distribution of ***n***-dimensional unit hypercube; for example, the sequence for two dimensions as shown in *Figure 1*(a)-(b). Haverkamp constitutive functions have 8 input parameters that must be tested, thus, 8 dimensions were required for each matrix. The f_o^2 is given by $((1/N)\sum_{m=1}^{N} y_A^{(m)})^2$. The $y_{c_i}^{(m)}$ is also model output; all the dimensions in the matrix were taken from matrix A, except *i* column, that is, dimension, which was taken from matrix B.

The number of rows in the matrix indicates the number of simulation runs required, while the number of columns in the matrix indicates the number of input parameters to be tested. In this study, we have N=15,000 and k= 8 columns. To solve Eqs. (8) and (9), we need two matrix (A and B), i.e. 2N and k input parameters of N for each input parameter, that is, kN. In total, we have to simulate $N(k+2)$=15,000 (8+2)=15,000 runs. A higher N value would result in better estimation of sensitivity indices. Nossent *et al*. (2011) had demonstrated that an N value of 12,000 for 26 input parameters is sufficient to obtain reliable estimation. All these were coded in Simply FORTRAN software.



(a) (b)

Figure 1: First (a) 1,000 points and (b) 10,000 points of a two-dimensional Sobol' quasi-random sequence are generated for dimension 1 and 2.

Numerical Solution on Richards' Equation and Input Parameter Values

There are basically three types of Richards' equation: (1) θ_L-based; (2) ψ_m-based and (3) mixed.

In this study, we limit to θ_L-based, as follows:

$$\frac{\partial \theta_L}{\partial t} = \frac{\partial}{\partial z}\left[\left(K\frac{\partial \psi_m}{\partial \theta_L}\right)\frac{\partial \theta_L}{\partial z} - K\vec{k}\right] \tag{10}$$

where θ_L is volumetric water content (m^3 m^{-3}); t is time of simulation (s); z is vertical distance of simulation (m); K is hydraulic conductivity of the medium (m s^{-1}); ψ_m is matric pressure head (m); and $\bar{k}$ is vector unit with a value of positive one when it is vertically downwards. Its advantages and disadvantages are discussed in Celia *et al.* (1990). The limitation of θ_L-based governing equation has been properly addressed, and its preliminary results have been discussed by Goh and Noborio (2013).

Eq. (10) was solved numerically using finite difference method with cell-centered and fully implicit for spatial and temporal discretization, respectively. The solution was implemented using Simply FORTRAN software. The algebra solution to Eq. (10), is as follows:

$$\frac{\theta_{L(k)}{}^{n+1}-\theta_{L(k)}{}^{n}}{\Delta t} = \frac{K_{k+\frac{1}{2}}\left(\frac{\partial \psi_m}{\partial \theta_L}\right)_{k+\frac{1}{2}}}{\Delta z_k(0.5\Delta z_{k+1}+0.5\Delta z_k)}\left(\theta_{L(k+1)}{}^{n+1} - \theta_{L(k)}{}^{n+1}\right) - \frac{K_{k-\frac{1}{2}}\left(\frac{\partial \psi_m}{\partial \theta_L}\right)_{k-\frac{1}{2}}}{\Delta z_k(0.5\Delta z_k+0.5\Delta z_{k-1})}\left(\theta_{L(k)}{}^{n+1} - \theta_{L(k-1)}{}^{n+1}\right) - \frac{K_{i,j,k+\frac{1}{2}}\vec{k} - K_{i,j,k-\frac{1}{2}}\vec{k}}{\Delta z_k} \quad (11)$$

where k indicates a cell-centered number in z-direction in cartesian coordinate system; Δt (s) is time-step size; $\theta_{L(k)}{}^{n}$ (m^3 m^{-3}) and $\theta_{L(k)}{}^{n+1}$ (m^3 m^{-3}) indicates volumetric water content at old time level (n) and new time level (n+1), respectively; $K_{k+1/2}$ (m s^{-1}) is hydraulic conductivity at the interface between cell k and k + 1; $K_{k-1/2}$ (m s^{-1}) is hydraulic conductivity at the interface between cell k-1 and k; $(\partial\psi_m / \partial\theta_L)_{(k+1/2)}$ is partial derivative of ψ_m with respect to θ_L at the interface between the cell k and k + 1; $(\partial\psi_m / \partial\theta_L)_{k-1/2}$ is partial derivative of ψ_m with respect to θ_L at the interface between the cell k - 1 and k; Δz_{k+1} (m), Δz_k (m) and Δz_{k-1} (m) correspond to the spatial sizes of spacing of cell k + 1, k and k - 1. $\theta_{L(k+1)}{}^{n+1}$ (m^3 m^{-3}), $\theta_{L(k)}{}^{n+1}$ (m^3 m^{-3}) and $\theta_{L(k-1)}{}^{n+1}$ (m^3 m^{-3}) are the volumetric water content at new time level of cell k +1, k and k - 1, respectively.

An iterative method was used to solve the mathematical algebra of Eq. (10) (Tu *et al.* 2007). A convergence factor criterion was used to indicate the condition for iteration termination process, that is, absolute maximum difference $|\theta_{L(k)}{}^{(n+1)} - \theta_{L(k)}{}^{n}|$ for every single cell.

The constitutive functions implemented were (Haverkamp *et al.* 1977):

$$\psi_m = -10^{-2} exp\left[\frac{\alpha(\theta_s-\theta_r)}{\theta_L-\theta_r} - \alpha\right]^{\frac{1}{\beta}} \quad (12)$$

$$K = K_s \frac{A}{A+(-100\psi_m)^B} \quad (13)$$

where α, β, A and B are fitting parameters; θ_r (m^3 m^{-3}) is residual volumetric water content; θ_s (m^3 m^{-3}) is saturated volumetric water content; and K_s (m s^{-1}) is saturated hydraulic conductivity.

Base case values or default values used in the current simulation were from Haverkamp *et al.* (1977) for yolo light clay. Numerical input parameters, that is, time-step size and spatial discretization size were from Goh and Noborio (2013). These input values are tabulated in Table 1. The time-step size (500 s), spatial discretization size (1 cm) and convergent value (10^{-12} m^3 m^{-3}) were set after considering mass balance ratio and iteration number. The upper boundary condition was set at 0.495 m^3 m^{-3}. The simulations of water infiltration profile at 10^5, 10^6 and $3\text{x}10^6$ s are shown in *Figure 2*. The simulations have shown comparable results to the published data of Philip's semi-analytical solution provided by Haverkamp *et al.* (1977).

TABLE 1
The default values of input parameters from Haverkamp *et al*. (1977) for yolo light clay

Parameter	Value
α	739
θ_r	0.124 m^3 m^{-3}
θ_s	0.495 m^3 m^{-3}
β	4
A	124.6
B	1.77
K_s	1.23x10^{-7} m s^{-1}
$\theta_L(initial)$	0.2376 m^3 m^{-3}

Note: θ_r is residual volumetric water content, θ_s is saturated volumetric water content, K_s is saturated hydraulic conductivity, θ_L (initial) is initial value of volumetric water content, and *α, β, A* and *B* are fitting coefficients (Eqs. (12) and (13)).

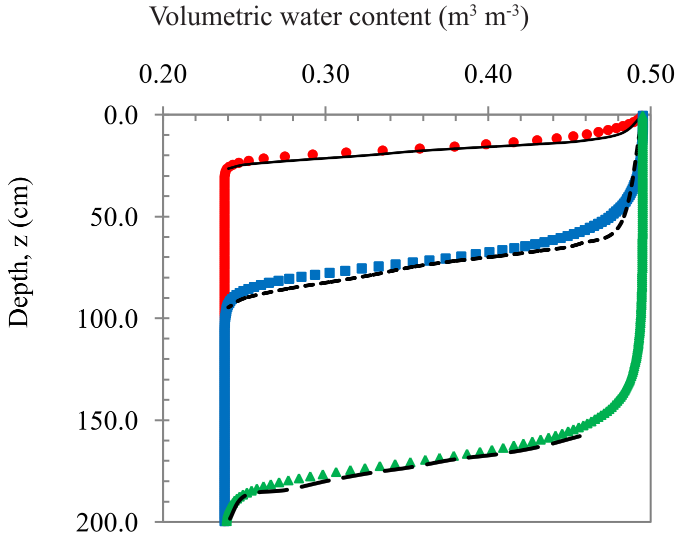


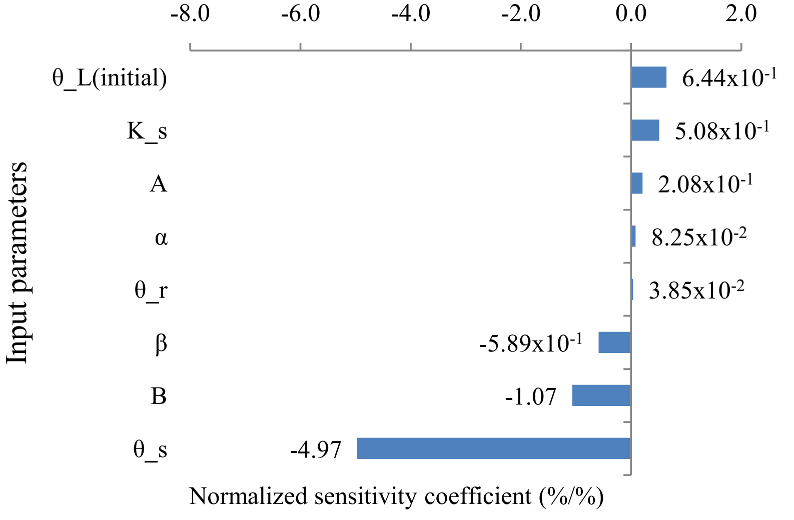
Figure 2: Simulated water infiltration profile at 10^5 s (●), 10^6 s (■) and 3x10^6 s (▲), and in comparison with published data of Philip's semi-analytical solution at 10^5 s (———), 10^6 s (-----) and 3x10^6 s(— —) retrieved from Haverkamp et al. (1977).

Sensitivity Analysis
Local and global sensitivity analyses were carried out on Haverkamp constitutive functions implemented in Richards' equation based on the input parameters shown in Table 1. Both analyses were studied at 10^5 s simulation time. Local sensitivity analysis was carried out to determine normalized sensitivity coefficients by varying input parameters at prescribed percentage, while global sensitivity analysis was

carried out to determine the fraction of model output variance, that is, first order index (S_i) and total effect index (S_{T_i}), for each parameter, based on uncertainty in input value range.

Local Sensitivity Analysis

Input parameters were varied ± 5 % as recommended by Zheng and Bennett (2002), and the results are shown in *Figure 3*. Apart from positive and negative relations as shown in the respective upper and lower parts of the figure, parameter θ_L (initial) appeared to have the most sensitivity in positive relations, and this was followed by parameters K_s, A, α and θ_r in decreasing order. In negative relations, parameters θ_s, B and β are in decreasing order of importance.



Note: θ_r is residual volumetric water content, θ_s is saturated volumetric water content, K_s is saturated hydraulic conductivity, θ_L(initial) is initial value of volumetric water content, and α, β, A and B are fitting coefficients (Eqs. (12) and (13)).

Figure 3: Normalized sensitivity coefficient (%/%) from local sensitivity analysis.

Global Sensitivity Analysis

The uncertainty in input parameters could be categorized based on types of distribution. For example in Fox *et al*. (2010), normal, uniform, triangular and lognormal distributions were used. In this study, uniform distribution was used for 8 input parameters, and it was based on significant digit approximation, as in Table 2. Similarly, uniform distribution was also used by Saltelli *et al*. (2004) for 103 parameters, Yang (2011) for 5 parameters, and also for Campolongo *et al.* (1999) and Morris (1991). This was followed by uncertainty propagation where the uncertainty in input parameters' values were translated into model output variance. The results are displayed in *Figure 4* for each parameter investigated.

Quantitative contribution for each parameter to model output variance is given by first order index (S_i) and total effect index (S_{T_i}).

TABLE 2

Uncertainty data range for input parameters

Parameter	Data range
α	738.5 – 739.499
θ_r	0.1235 – 0.124499
θ_s	0.495 – 0.495499
β	3.5 – 4.499
A	124.55 – 124.6499
B	1.765 – 1.77499
K_s	4.4275×10^{-2} – 4.428499×10^{-2}
$\theta_L(initial)$	0.23755–0.2376499

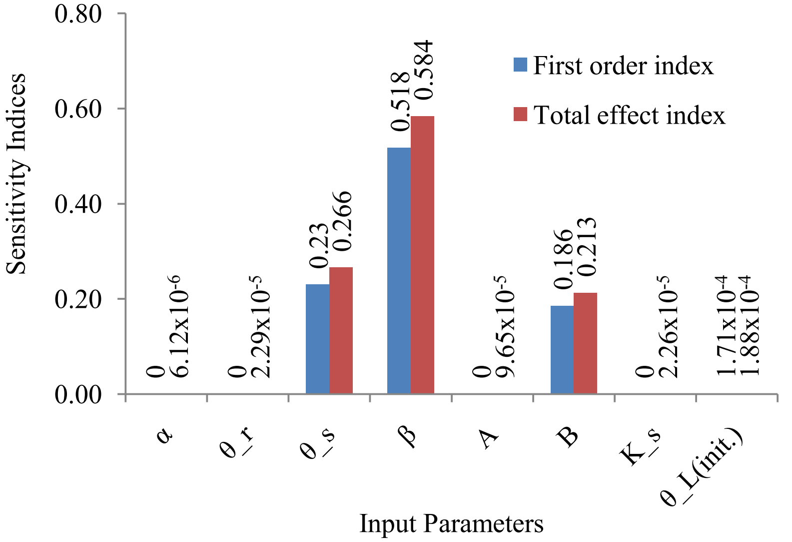
Note: Refer to Table 1 for parameters' definitions.

In water infiltration front, β, θ_s and B were the most influencing parameters for the model output variance. The $S_\beta = 0.5177$ represents partial variance of parameter β to total model output variability, while the additive contribution of 8 input parameters gave $\sum_{i=1}^{8} S_i = 0.9334$. This suggests that the first order sensitivity index of parameter β explained 51.77 % of total model output variability, whereas a 93.34 % of total model variability was captured by 8 parameters. The first order index was then followed by parameters θ_s and B at 22.98 % and 18.58 %, respectively. The sum of first order index of the three most important parameters explained 93.32 % of the variability in the output. The presence of balance given by $1 - \sum_{i=1}^{8} S_i$, suggests interaction effects.

The total effect sensitivity index in Figure 4 shows that the S_{T_i} has a consistent trend as the S_i, that is, the ranking of parameters by S_i values agreed well with those from S_{T_i} values. It was noted that all the S_{T_i} values were greater than S_i values. This again suggests each parameter interacts as a pair, or more with the other parameters. $S_{T_i} - S_i$ was used to estimate the interaction effect index of second order and/or higher order effects. The highest interaction effect was showed by $S_{T_\beta} - S_\beta$, yielding 6.63 %, followed by parameters θ_s and B with corresponding values of 3.62 % and 2.74 %. The interaction effect shown by $S_{T_\beta} - S_\beta$ suggests that this value was the result of interaction between β and other parameters, but does not specifically indicate which other parameters are involved and what degree of interaction is present. As stated by Saltelli and Annoni (2010), the purpose of S_i is to determine the acquirable expected variance reduction if X_i could be fixed, whereas S_{T_i} is the expected variance remaining if all parameters but X_i could be fixed. In the current study as we were only interested in identifying the important parameters responsible for model output variability, the estimations of S_i (Eq. (8)) and S_{T_i} (Eq. (9)) were sufficient without the need to identify the interaction effect index.

The parameters α, θ_r, A, K_s and θ_L (*initial*) were identified as unimportant as both S_i and S_{T_i} were low in values. Although there were values given by S_{T_i} -

S_i for these parameters, they were considered negligible when compared to *β*, θ_s and *B*. Therefore, the model output variability was not significantly affected by the parameters' uncertainty range, and thus, they could be termed as minor parameters, based on the uncertainty ranges of parameters tested. Besides, it also suggests that the significant digits of *β*, θ_s and *B* should be increased in order to reduce the uncertainty range that resulted in high variability in the model output.



Note: *θ_r* is residual volumetric water content, *θ_s* is saturated volumetric water content, *K_s* is saturated hydraulic conductivity, *θ_L(init.)* is initial value of volumetric water content, and *α, β, A* and *B* are fitting coefficients (Eqs. (12) and (13)).

Figure 4: Sensitivity indices versus input parameters. The first order index and total effect index are represented by S_i and S_{T_i}, respectively.

In addition to S_i and S_{T_i}, uncertainty propagation also allowed estimation of other statistical parameters; for instance, mean, median, 95% confident interval, and so on, as tabulated in Table 3. Our results showed a 95% confident interval of volumetric water content range between 0.302 and 0.313. A slight increase in value of median in comparison to mean indicates negative skew of the model output distribution. While von Hippel (2005) has proven that these two parameters are inferior indicators for skewness in some cases of distribution, other statistical parameters verified the negative skew such as a slight negative value of skewness, i.e. -0.034 (a left-skewed), which is also illustrated in *Figure 5*. An exact normal distribution would have an exact value of 3 for kurtosis, and thus, a value of 2.676 for kurtosis indicates a slight characteristic of platykurtic, that is, lower peak than normal distribution and lighter tails.

The comparisons between local and global sensitivity analyses have shown some differences. The local tool was able to rank parameters from important to unimportant. These parameters were determined by a linear parameter change, one-at-a-time (OAT) by keeping other parameters constant, which means all other input spaces were not fully studied. Uncertainty estimation on model output could still be carried out in local tool by multiplying percentage change in input parameter to normalized sensitivity coefficient, as demonstrated by Goh and Noborio (2013); however, sensitivity indices for first order, total effect index, interaction effects index, and statistic parameters, as in Table 3, would be unavailable.

TABLE 3
Uncertainty analysis on statistical parameters for model output probability distribution obtained from the Sobol' variance-based method.

Statistical parameters	Values
Mean	0.30758
Median	0.30762
95 % Confident Interval	0.302 - 0.313
Standard Deviation	3.489×10^{-3}
Standard Error of Mean	2.848×10^{-5}
Minimum	0.29406
Maximum	0.31751
Skewness	-0.034
Kurtosis	2.676

Note: Results were based on 100,000 s simulation time, and 18.5 cm depth from ground surface.

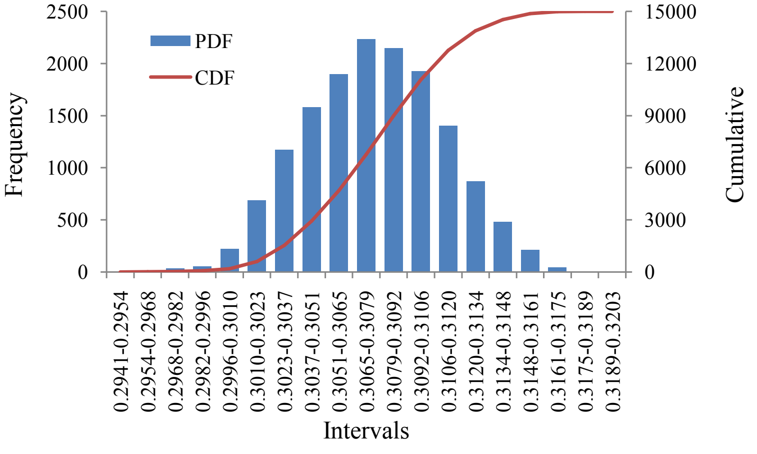
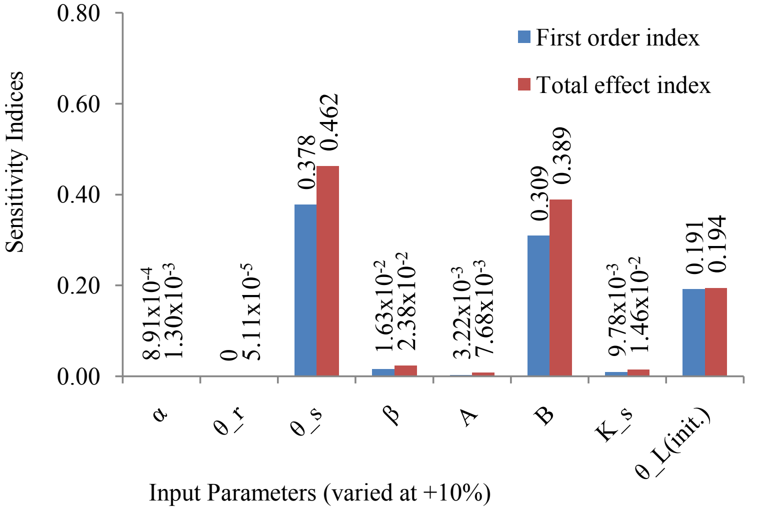


Figure 5: Probability (left y-axis) and cumulative (right y-axis) output distribution functions obtained via Sobol' variance-based method.

In addition, we observed consistency in parameter ranking for first order and total effect index on input parameters, after variation of +10% as shown in *Figure 6*, with local sensitivity tool results in *Figure 3*. Although both local and global sensitivity tools illustrated a similar trend of parameter ranking, in reality the range of uncertainty on each input parameter does not have equal percentage of variation, for instance in Fox *et al.* (2010). Under such conditions of different percentage variation between input parameters and previously unknown presence of interaction effects between input parameters at unexplored input spaces, global sensitivity analysis is a very attractive and robust tool.



Note: θ_r is residual volumetric water content, θ_s is saturated volumetric water content, K_s is saturated hydraulic conductivity, θ_L(init.) is initial value of volumetric water content, and α, β, A and B are fitting coefficients (Eqs. (12) and (13)).

Figure 6: Sensitivity indices versus input parameters varied at +10%. The first order index and total effect index are represented by S_i and S_{T_i}, respectively.

CONCLUSION

The global sensitivity analysis was shown to have a better coverage of input spaces, more informative indicators than the local sensitivity tool, and was able to provide statistical description on the model outputs. This study also found that parameters β, θ_s and B were dominant parameters as they had substantially greater first order index and total effect index than other parameters. As a result, this suggests that increasing significant digits of these parameters with narrower uncertainty range would be able to reduce their influence on model output variability. Total effect index was found to be slightly greater than first order index for every parameter, suggesting that interaction effects between parameters of second order, higher order and so forth were not as important as first order index. Although local sensitivity analysis is capable of parameter ranking and uncertainty analysis by varying a single parameter at a time, global sensitivity analysis would be a better alternative as it is based on statistical theory and in addition to those capabilities of local sensitivity analysis, it can estimate interaction effects between parameters and also provide some informative statistical analysis on model outputs distribution such as 95% confidence interval, skewness, kurtosis and other statistical parameters.

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